# Shape Decomposition for Graph Representation

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**Abstract.** The problem of shape analysis has played an important role in the area of image analysis, computer vision and pattern recognition. In this paper, we present a new method for shape decomposition. The proposed method is based on a refined morphological shape decomposition process. We provide two more analysis for morphological shape decomposition. The first step is scale invariant analysis. We use a scale hierarchy structure to find the invariant parts in all different scale level. The second step is noise deletion. We use graph energy analysis to delete the parts which have minor contribution to the average graph energy. Our methods can solve two problems for morphological decomposition – scale invariant and noise. The refined decomposed shape can then be used to construct a graph structure. We experiment our method on shape analysis.

## 1 Introduction

Shape analysis is a fundamental issue in computer vision and pattern recognition. The importance of shape information relies that it usually contains perceptual information, and thus can be used for high level vision and recognition process. There has already many methods for shape analysis. The first part methods can be described as statistical modeling [4] [12][9] [11]. Here a well established route to construct a pattern space for the data–shapes is to use principal components analysis. This commences by encoding the image data or shape landmarks as a fixed length long vector. The data is then projected into a low-dimensional space by projecting the long vectors onto the leading eigenvectors of the sample covariance matrix. This approach has been proved to be particularly effective, especially for face data and medical images, and has lead to the development of more sophisticated analysis methods capable of dealing with quite complex pattern spaces. However, these methods can't decompose the shapes into parts and can't incorporate high level information from shape. Another problem which may hinder the application of these method is that the encoded shape vectors must be same length which need large human interaction pre-processing.

Another popular way to handle the shape information is to extract the shape skeleton. The idea is to evolve the boundary of an object to a canonical skeletal form using the reaction-diffusion equation. The skeleton represents the singularities in the curve evolution, where inward moving boundaries collide. With the

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skeleton to hand, then the next step is to devise ways of using it to characterize the shape of the original object boundary. By labeling points on the skeleton using so-called shock-labels, the skeletons can then be abstracted as trees in which the level in the tree is determined by their time of formation[15,8]. The later the time of formation, and hence their proximity to the center of the shape, the higher the shock in the hierarchy. The shock tree extraction process has been further improved by Torsello and Hancock[16] recently. The new method allows us to distinguish between the main skeletal structure and its ligatures which may be the result of local shape irregularities or noise. Recently, Bai, Latecki and Liu [1] introduced a new skeleton pruning method based on contour partition. The shape contour is obtained by Discrete Curve Evolution [10]. The main idea is to remove all skeleton points whose generating points all lie on the same contour segment. The extracted shape skeleton by using this method can better reflect the origin shape structure.

The previous two shape analysis methods, statistical modeling and shape skeletonization, can be used for shape recognition by combining graph based methods. For example, Luo, Wilson and Hancock [2] show how to construct a linear deformable model for graph structure by performing PCA(Principal Component Analysis) on the vectorised adjacency matrix. The proposed method delivers convincing pattern spaces for graphs extracted from relatively simple images. Bai, Wilson and Hancock[18] has further developed this method by incorporating heat kernel based graph embedding methods. These method can be used for object clustering, motion tracking and image matching. For the shape skeleton methods, the common way is to transform the shape skeleton into a tree representation. The difference between two shape skeletons can be calculated through the edit distance between two shock trees[16].

Graph structure is an important data structure since it can be used to represent the high level vision representation. In our previous work [19], we have introduced an image classifier which can be used to classify image object on different depictions. In that paper, we have introduced an iterative hierarchy image processing which can decompose the object into meaningful parts and hence can be used for graph based representation for recognition.

In this paper, we will introduce a new shape decomposition method. Our method is based on morphological shape decomposition which can decompose the binary shapes through iterative erosion and dilation process. The decomposed parts can then be used to construct a graph structure i.e. each part is a node and the edge relation reflect the relationship between parts, for graph based shape analysis. However, morphological shape decomposition has two shortcomings. First, the decomposition is not scale invariant. When we change the scale level for the same binary shape the decomposition is different. Second, the decomposed parts contains too much noise or unimportant parts. When we use graph based methods for shape analysis these two problems will certainly produce bad influence for our results. Our new method provide two more analysis for morphological decomposition. We first solve the scale invariant problem. We decompose the shape through a hierarchy way. From top to bottom each level representing a different scale size for the same binary shape from small to big. We decompose each level through morphological decomposition and then find the corresponding parts through all levels. We call these parts invariant in all scale levels and use them to represent the binary shapes. The second step is used to delete the noise parts which are normally unimportant and small. We construct the graph structure for the decomposed parts and use graph energy method to analysis the structure. We find the parts(nodes) which has minor or none contribution to the average energy for the whole graph structure. The rest parts are kept as important structure for the shape.

In Section 2, we first review some preliminary shape analysis operations i.e. the tradition morphological shape decomposition. In Section 3, we describe a scale invariant shape parts extraction method. In Section 4, we will describe our graph energy based noise deletion and in Section 5 we provide some experiment results. Finally, in Section 6, we give conclusion and future work.

## 2 Background on Morphological Shape Decomposition

In this section, we introduce some background on shape morphology operation. Morphological Shape Decomposition (MSD)[14] is used to decompose the shape by the union of all the certain disks contained in the shape. For a common binary shape image, it contains two kinds of elements "0"s and "1"s, where "0" represents backgrounds and "1" represents the shape information. The basic idea of morphology in mathematics can be described as below

$$(M)_u = \quad m + u \; m \in M \tag{1}$$

. There are two basic morphological operations, the dilation of $M$ by $S$ and the erosion of $M$ by $S$, which are defined as follows:

$$M \oplus S = \bigcup_{s \in S} (M)_S \tag{2}$$

and

$$M \ominus S = \bigcup_{s \in S} (M)_{-S} \tag{3}$$

. There are also two fundamental morphological operation based on dilation and erosion operations, namely the opening of $M$ by $S(M \circ S)$ and closing of $M$ by $S(M \bullet S)$. The definitions are given below:

$$M \circ S = (M \ominus S) \oplus S \tag{4}$$

$$M \bullet S = (M \oplus S) \ominus S \tag{5}$$

A binary shape $M$ can be represented as a union of certain disks contained in $M$

$$M = \bigcup_{i=0}^{N} L_i \oplus iB \tag{6}$$

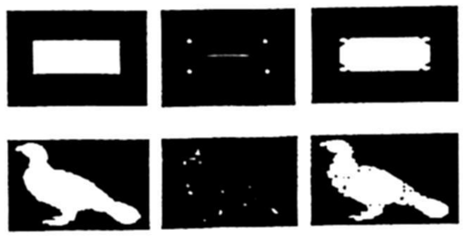

**Fig. 1.** An example for morphological shape decomposition.

where $L_N = X \ominus NB$ and

$$L_i = (M(\bigcup_{j=i+1}^{N})) \ominus iB \quad 0 \le i < N \tag{7}$$

$N$ is the largest integer which satisfy

$$M \ominus NB \neq \oslash$$

it can be computed by an iterative shape erosion program. B is defined as morphological disks. We call $L_i$ loci and $i$ as corresponding radii. We follow the work by Pitas and Venetsanopoulos [14] to compute the $L_i$ and $i$. This can give us an initial shape decomposition.

An example is shown in Figure1. Here two shapes(the left column) are given, in which a rectangular shape can be decomposed into five parts. In the upper-middle column of Figure 1 there are one center part and four corners. However, different with the normal shape representation which contains two elements, 0s and 1s, the loci part is represented by the elements of $i$ and the backgrounds are still 0. It is called "Blum Ribbon". With this representation at hand, we can reconstruct the origin shape[14]. The right column in this figure shows the reconstructed shapes by using the "Morphological Ribbon".

## 3 Scale Invariant Structure Extraction

In the introduction part, we have emphasized the importance of incorporating graph structure representation with shape analysis. It is normal to construct a graph structure from morphological shape decomposition. We can simply treat each part as a node in the graph and the edge relationship is deduced from the adjacency between each pair of parts. If two parts are adjacent or overlap then the weight between the two corresponding nodes are non-zero. In this paper, we dilate the parts with the disk radius size two more than the origin eroded

**Fig. 2.** Graph structure example from morphological shape decomposition.

skeleton. For example, if two parts $I$ and $J$'s radius are $r_i$ and $r_j$ with $I$ and $J$ the corresponding loci, we first dilate these two parts by the radius $r_i + 2$ and $r_j + 2$. Then the weight between parts $I$ and $J$ is $and(I \ominus (r_i + 2)\ J \oplus (r_j + 2))/or(I \oplus (r_i + 2)\ J \oplus (r_j + 2))$ which reflect both the overlap and adjacent relationship. We can use a five nodes graph to represent the rectangular shape 2 while the center is connected with four corners.

However, the graph structure constructed from this morphological based shape decomposition method is not suitable for graph based shape analysis. It is sensitive to scaling, rotation and noise [7]. An example is shown in Figure 3 here we decompose a set of different size rectangular , we can observe two things 1) It doesn't satisfy scale invariant. As we can see, when the scale is different the decomposition results is different. At the small scale level, the rectangular shape decomposed skeleton include one line and four small triangles. While at large scale level, the skeleton include one line and twelve triangles.

### 3.1 Hierarchy Morphological Decomposition

We propose a solution which is to decompose the shape in different scale and find the corresponding matching parts to represent the shape. The idea is when a shape is given, we squeeze and enlarge the shape image in a sequence list. We decompose this sequence image shapes. We then find the corresponding parts for this sequence shape decomposition. The stable scale invariant shape decomposition is then found by choose the parts which appear in all different scale levels.

In Figure 3, we still use the example of the rectangular, we first squeeze and enlarge the shape by 15 percent each time. We choose three squeezed and three enlarged shapes – altogether we have five shapes. We then decompose this sequence through morphological decomposition described in the previous section. We then find the correspondence in a hierarchy style. From the correspondence results, we notice that the parts which appear in all levels are the center line and four dots in the corners. The proposed methods can solve the scale invariants

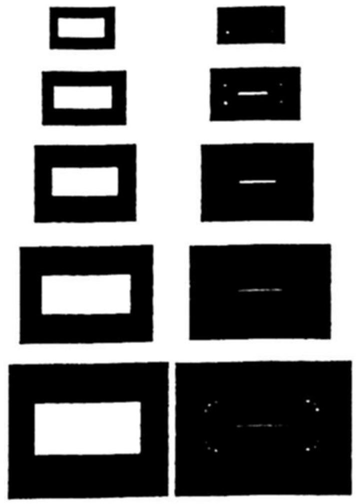

**Fig. 3.** Example for the same shape morphological decomposition in different scale.

problem for shape decomposition. Like SIFT feature [5], we consider the shape decomposition through a hierarchy way.

## 4 Graph Energy based Noise Deletion

We continue to use the idea from spectral graph theory [3] to delete the noise in morphological shape decomposition. Our idea is to use graph Laplacian energy which reflect the connectiveness and regularity for the graph to delete the parts(nodes) which has minor or none contribution to the average graph Laplacian energy per node. The solution is to iteratively delete the parts and finally stop this process when the average graph Laplacian energy per node never rise.

We first review the graph Laplacian energy [6]. The Laplacian matrix is defined as $L = D - A$, in which $D$ is a degree matrix, and $A$ an adjacency matrix. Laplacian graph energy has the following standard definition: for a general graph $G = (V\ A)$, with arc weights $w(i\ j)$ the Laplacian energy is

$$\mathcal{E}(G) = \sum_{i=1}^{|V|} \left| \lambda_i - 2\frac{m}{V} \right| \tag{8}$$

In which: the $\lambda_i$ are eigenvalues of the Laplacian matrix; $m$ is the sum of the arc weights over the whole graph, or is half the number of edges in an unweighted graph; $V$ is the number of nodes in graph. Note that $2m/V$ is just the average (weighted) degree of a node. Now, the Laplacian energy of a graph can rise or fall; our tests show that this rise and fall is strongly correlated with the variance in the degree matrix $D$. This means local minima tend to occur when the graph is regular.

Since we want to use graph Laplaican energy, we need to first construct a graph structure for morphological decomposed parts. The graph structure can be constructed through the method from previous section. We treat each parts from morphology decomposition as a node in the graph $G$, the edge relationship is found through the adjacency and overlap relationship between each pair of parts.

The process of noise deletion is listed below: 1) We compute the initial average graph energy for the initial state decomposition $\mathcal{E}(G)/N$. 2) For each iteration, we go through all the nodes in the graph $G$. For each node we judge whether we should delete this node. We just compare the previous average graph energy $\mathcal{E}(G)/N$ with the average graph energy with this node deleted $\mathcal{E}(G_{di})/N-i$, where $G_{di}$ is the graph with $i$th nodes deleted. If the the average graph energy $\mathcal{E}(G_{di})/N-i$ is larger than the previous average energy then we should delete this node and update the graph structure $G$. 3) Repeat step two, until $\mathcal{E}(G_{d}i)/N-i$ never rise. 4) Output the final decomposition.

The previous process can detect the nodes which has weak link with rest nodes in the graph. It will prune the graph structure until it near or reach regular while keep strong connectiveness within the rest nodes.

## 5 Experiment

In this section, we provide some experiment results for our methods. Our process can be simply described as below:

- For a given binary shape, we first squeeze and enlarge it to construct the hierarchy scale levels from small to big.
- Perform morphological shape decomposition for each level, in this paper we use Pitas and Venetsanopoulos [14] method. Find the corresponding matching parts through all levels. These parts input for the next step.
- Use the output from last step to construct the graph structure. Use average graph energy method to delete the noise nodes(parts) in the graph. Repeat this step until the average graph energy never rise. Output the final graph structure.

We experiment on shock graph database which composed of 150 silhouettes of 10 kinds of objects [16]. An example of database is shown in Figure 4.

In Figure 5, we give some results for our methods, here in the left column is the origin shape, the middle column is the pruned skeleton parts from morphological shape decomposition and the right column is the re-constructed shape by using the skeleton centers in the middle column. From this example, we can see that our algorithm can reduce some noise parts from the origin morphological

**Fig. 4.** Sample views of the silhouette objects

decomposition while keep the important parts. It can be seen that the reconstructed shapes are quite similar to the original shapes and thus keeps the most important information for further analysis.

In table 1 we listed the variation for the number of parts within the same class for tradition morphological shape decomposition method(MSD) and our method. It is clear that the variations for the number of parts for the tradition morphological shape decomposition is higher than our method.

**Table 1.** Variation for the number of parts with different shape decomposition methods.

| Class Name | MSD | Our Method |
|---|---|---|
| Car | 8.5 | 4.1 |
| Children | 11.4 | 6.7 |
| Key | 9.0 | 5.0 |
| Bone | 8.5 | 4.7 |
| Hammer | 4.5 | 3.2 |

## 6 Discussions and Conclusions

In this paper, we proposed a new shape decomposition method which extended the morphological methods. It can conquer two problems for the current morphological methods, scale invariant and noise. We have proposed a graph Laplacian energy based hierarchy shape decomposition. We can extract more stable graph structure by using our methods. Our next step is to use these graph structures to do shape analysis. One possible way is to combine the spectral graph invariants [17] for shape recognition. Recently, Trinh and Kimia [13] has proposed a graph generative for shape through the analysis of shock graphs. We can also extend our methods with graph generative model for morphological decomposition.

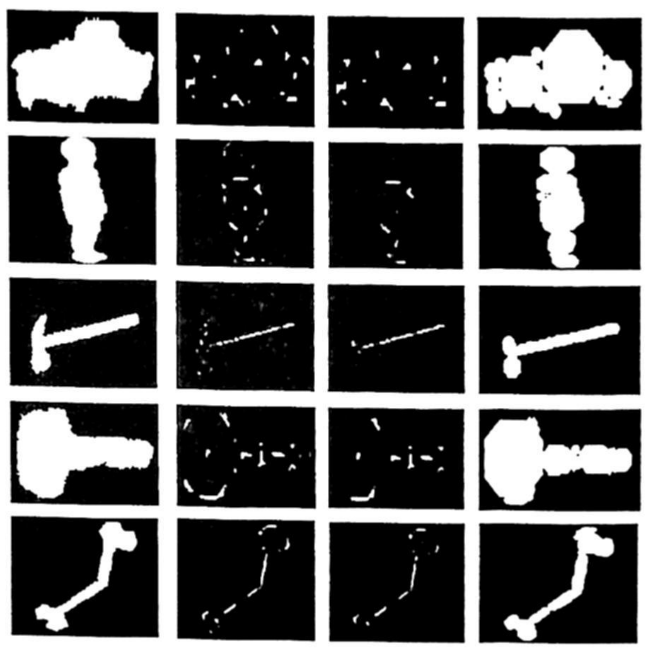

**Fig. 5.** Example for our methods

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